Inverse Hyperbolic Functions Encyclopedia Article

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The inverse hyperbolic **functions** are simply the inverse of the different **hyperbolic functions** cosh *x*, coth *x*, csch *x*, sech *x*, sinh *x*, and tanh *x*. The inverse of a function is that function reflected about the line $y = x$ and usually denoted by $f^{-1}(x)$. The inverse hyperbolic functions are denoted as $\sinh^{-1}x$ (or arcsinh *x*), $\cosh^{-1}x$ (or arccosh *x*), $\tanh^{-1}x$ (or arctanh *x*), $\text{csch}^{-1}x$ (or arccsch *x*), $\text{sech}^{-1}x$ (or arcsech *x*), and $\coth^{-1}x$ (or arccoth *x*). Since the hyperbolic functions are based on the exponential function it is not surprising that the inverse hyperbolic functions are based on the natural logarithmic function, ln.

Like the hyperbolic functions the inverse hyperbolic functions are periodic since they are elliptic functions. Corresponding to the two main hyperbolic functions are the inverses of these functions. They are inverse hyperbolic **sine**, $\sinh^{-1}x$, and inverse hyperbolic **cosine**, $\cosh^{-1}x$ and are defined as: $\sinh^{-1}x = \ln(x + (x^2 + 1))$ and $\cosh^{-1}x = \ln(x +/- (x^2 - 1))$. The inverse hyperbolic cosine has the +/- sign because it is not a one to one function, that is the **domain** either has to be restricted or the +/- sign has to be used. Inverse hyperbolic tangent is defined as: $\tanh^{-1}.x = 1/2 \ln((1 + x)/(1 - x))$. The three other inverse hyperbolic functions, inverse hyperbolic cotangent, inverse hyperbolic secant and inverse hyperbolic cosecant, can be described in terms of inverse hyperbolic tangent, inverse hyperbolic sine and inverse hyperbolic cosine: $\coth^{-1}x = \tanh^{-1} 1/x$, $\text{sech}^{-1}x = \cosh^{-1} 1/x$, and $\text{csch}^{-1}x = \sinh^{-1} 1/x$. Just as the hyperbolic functions have identities the inverse hyperbolic functions have similar identities and negative argument formulas. The negative argument formulas are some of the more useful of these relations.