Fundamental Theorem of Algebra Encyclopedia Article

Fundamental Theorem of Algebra

Contents

Fundamental Theorem of Algebra

The fundamental **theorem** of **algebra** is the statement that every polynomial with **complex numbers** as coefficients has a complex number as a root, or equivalently, that such a polynomial has n **roots**, where n is its degree, in the complex numbers, counted with multiplicitly (that is, double roots count double and so on).

While this can be checked for **polynomials** of degree up to four, using the formulas for the roots, this approach will not work for higher degrees. Some early authors seem to have taken the fundamental theorem of algebra for granted, without realizing it required a justification. Albert Girard (1629) might have been the first to call attention to the statement of the fundamental theorem of algebra but without trying to justify it. The famous mathematician and philosopher Gotthold Leibniz (1702) even doubted its validity. The first serious attempt at a **proof** was made by the French mathematician **Jean Le Rond D'Alembert** in 1746 but his proof was incomplete. The first correct proof was given by the great German mathematician Carl Frederich Gauss in 1799. Gauss then subsequently gave three other proofs and since then there has been many more different proofs. Stricly speaking, the fundamental theorem of algebra is really a theorem in **Analysis**, since its **truth** rests on the **continuity** properties of real and complex numbers. The algebraic content of the theorem has been made explicit by the theory of real closed fields developed by Emil Artin and Otto Schreier (1926).