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A sequence of numbers is said to be **arithmetic** if the difference between any two successive terms is the same. For example the sequence 1,3,5,7,9,... is arithmetic because the difference between any two consecutive terms is 2. This difference is usually called the common difference. Thus the common difference in the arithmetic sequence 2,6,10,14,... is 4. If the terms of an arithmetic sequence are added together, this sum is called an arithmetic series. So 1+3+5+7+... and 2+6+10+14+... are arithmetic series. Arithmetic series may be infinite or finite. The finite arithmetic series 1+3+5+7+9 has a sum of 25. There is a formula for computing the sum of a finite arithmetic series which is useful when the number of terms is large. The young Carl Freidrich Gauss (1777-1855), who would later become one of the greatest mathematicians of all time, is said to have discovered this formula when his elementary school teacher assigned the class the task of adding up all the **whole numbers** from 1 to 100. While the other students began the drudgery of adding the numbers one by one, Gauss suddenly realized that there was a simpler way. The sum 1+2+3+4+...+97+98+99+100 is an arithmetic series with common difference 1. Gauss noticed that the sum of 1+100=101, 2+99=101, 3+98=101, 4+97=101, and so on through 50+51=101. So, he reasoned, there are 50 pairs of numbers each of whose sum is 101; therefore, the sum of this series is 101x50=5050. Gauss, of course, finished before the other students and was the only one to get the correct answer. Notice that 101 is the sum of the first and last terms of the series and 50 is the number of pairs or 100/2. So if 101x50 is written as 101x100/2 it can be seen as a special case of the formula (first term + last term)x(the number of terms) divided by 2. In general, if the nth term of the series is designated by a_n, then $a_1 1+a_2+a_3+...+a_n=(n(a_1+a_n))/2$. If we try this out on 1+3+5+7+9, we get 5(1+9)/2=25, which agrees with our answer from summing the terms in the usual way.